Graph Theory
Lecture 12: Matching

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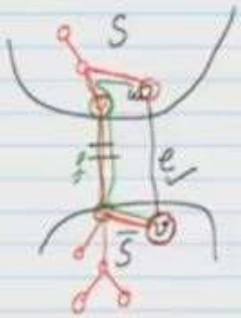
Lemma: Revision

Lemma

Let $S \subseteq V$ & $\bar{S} = V - S$.

Let e be the minimum cost edge connecting S and $\bar{S}$. Then e is part of MST.



Proof: Suppose we have a minimum spanning tree T not containing e. Then we prove the T is not a MST.

Let $e = (u, v)$, $u \in S$ & $v \in \bar{S}$. Since T is a spanning tree it contains an unique path from u to v which together with $e = (u, v)$ forms a cycle.

$cost(e) \leq cost(f)$

This green path has to include another edge f connecting S & $\bar{S}$.

$T + e - f$ is another spanning tree and has less cost than T.

So T is not MST.

Definition: Matching

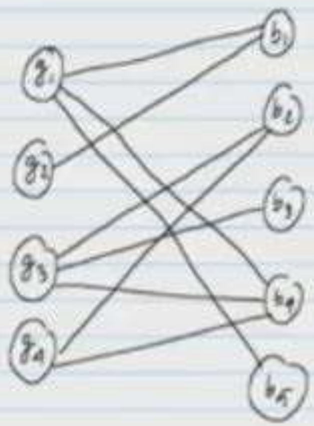
Matching

A matching M in G is a set of edges such that every vertex of G is incident to atmost one edge in M.

Ex Bipartite graph

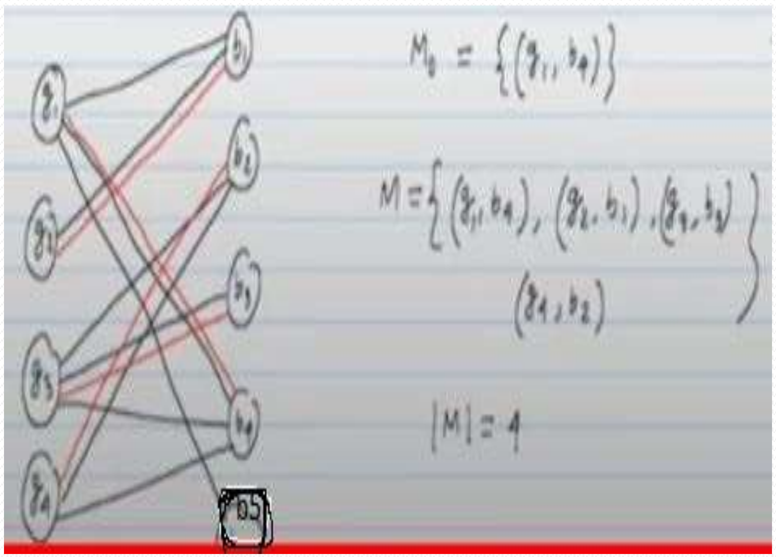
Suppose there are four girls $\{g_1, g_2, g_3, g_4\}$

" 5 boys $\{b_1, b_2, b_3, b_4, b_5\}$ represented by Graph G1



Graph G1

Example: Matching



Matching M (in red)in Graph G

Note:

M represents one of matching (Labelled in Red)

M_0 represents another matching.

M represents also Maximum Matching as this matching matches maximum possible girls with corresponding boys

Also size of matching M is 4.

Matching Terminology

The size of a matching is the number of edges in that matching

A matching is maximum when it has the largest possible size.

A perfect matching in a graph is matching that matches every vertex.